

# *Introduction to Quantum Computing*

**Dr. Edric Matwiejew**



**pawsey**



Meteorites by Wajarri Yamatji artist Margaret Whitehurst

# Acknowledgment of Country

***Ngaala Kaaditji Noongar moort keyen kaadak nidja boodja***

We acknowledge the traditional owners of land on which Pawsey is located, the Noongar Whadjuk People – their ancestors and elders, past, and present – as the original custodians of this land.

We pay our respects to the traditional owners of the land on which we meet today.



# Outline

- Introduction to the Pawsey Quantum Supercomputing Innovation Hub
- Quantum Computing:
  - Bits vs qubits, superposition, entanglement
  - Quantum circuits and quantum gates.
  - Quantum computing landscape: NISQ devices, analog vs digital
- Quantum Algorithms:
  - Quantum parallelism
  - Adiabatic quantum computing
  - Hamiltonian Simulation
  - Variational quantum circuits (VQCs)
- Hands-On:
  - Programming quantum circuits with Qiskit
  - Solving the max-cut problem with QAOA

*Quantum Computing at Pawsey*



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# Pawsey Supercomputing Research Centre



- National High Performance Computing Centre headquartered in Perth, Western Australia on Whadjuk Noongar country
- Launched as Pawsey in 2014, UJV foundations back to 2000
- 50+ Staff employed via CSIRO, national science agency
- AU\$70m capital refresh by Australian Government
- Houses the fastest Supercomputing in Australia
- **AU\$5m NCRIS investment in developing a Quantum Supercomputing Innovations Hub, part of the National Quantum Strategy**

*Pawsey strives to be at the cutting-edge of technology. Our domain is scientific computing: accelerating discovery through world-class supercomputing infrastructure and expertise.*

The Pawsey Supercomputing Research Centre is an unincorporated joint venture between

and proudly funded by

Core Members



Founding Associate Member



# Quantum SUPERcomputing Innovation Hub

- Pawsey is on the cutting edge of computing technologies
- The Hub, supported by NCRIS funding, a collaborative platform uniting quantum & classical computing to drive scientific breakthroughs.
- Setonix-Q merit-allocation schemes provides access to quantum computing hardware and QC-dedicated classical compute.





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Australian-made,  
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computing**

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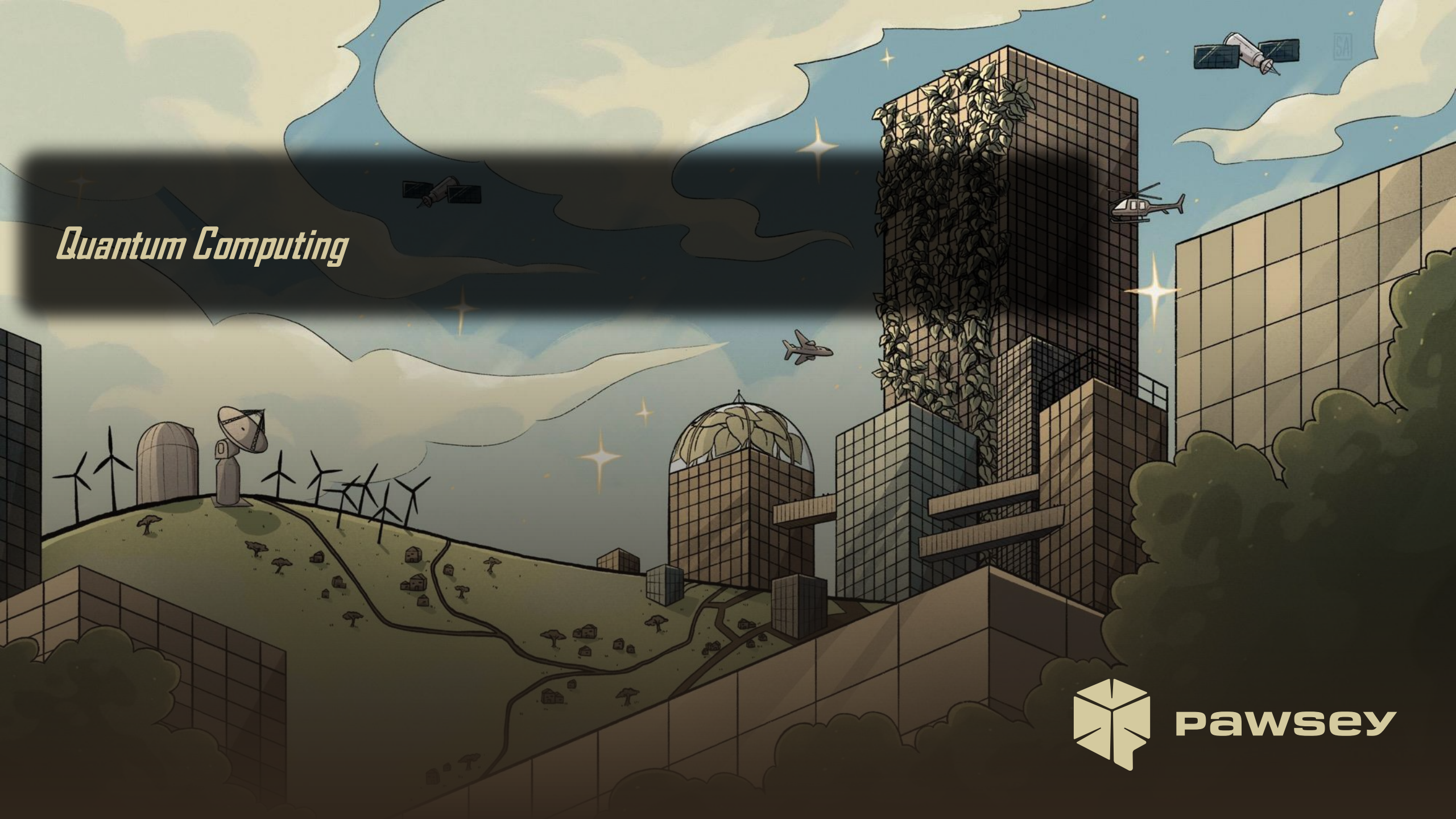
**Timeline:**

- February 2023: First quantum circuit demonstration
- May 2022: First quantum circuit demonstration
- August 2021: First quantum circuit demonstration
- January 2020: First quantum circuit demonstration
- July 2019: First quantum circuit demonstration
- May 2018: First quantum circuit demonstration
- March 2017: First quantum circuit demonstration



The Team

*Quantum Computing*



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# Quantum Computing

- Computation on information encoded in quantum states – typically **qubits**.
- Leverage quantum phenomena (superposition, interference and entanglement) to solve tough problems faster than classical computers.
- Quantum hardware and algorithms differ fundamentally from classical approaches.
- Potential for major speedups—though not for every problem.
- The challenge of realising practical quantum computing spans physics, mathematics, engineering, and computer science.
- No widely adopted high-level languages - programming with quantum “assembly” or “machine-level” instructions (e.g. laser pulse shaping)
- *Requires rethinking how problems are formulated and tackled, in a rapidly evolving field.*



# The Computational Basis

- A quantum computer uses the quantised states of a quantum object.
- Atomic energy levels, photon polarisation, molecular spins, or any other physical system that can be precisely controlled.
- In principle this quantum object can have any number of basis states.
- Most commonly the fundamental unit of information is a “qubit” (two basis states, often referred to as the **computational basis**):

$$\left\{ |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$



# The Computational Basis

Quantum basis states combined according to the **tensor product**  $\otimes$ :

$$|0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} (1) \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ (0) \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = |00\rangle$$

$$|0\rangle \otimes |1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} (1) \times \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ (0) \times \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = |01\rangle$$

# The Computational Basis

- An  $n$ -qubit **computational basis state** is often represented using a single base-10 integer.

$n = 2$ :

$$\begin{aligned} |0\rangle \otimes |0\rangle &= |00\rangle = |0\rangle \\ |0\rangle \otimes |1\rangle &= |01\rangle = |1\rangle \\ |1\rangle \otimes |0\rangle &= |10\rangle = |2\rangle \\ |1\rangle \otimes |1\rangle &= |11\rangle = |3\rangle \end{aligned}$$



# The Computational Basis

- An  $n$ -qubit **computational basis state** is often represented using a single base-10 integer.

$n = 3$ :

$$\begin{aligned}
 |0\rangle \otimes |0\rangle \otimes |0\rangle &= |000\rangle = |0\rangle \\
 |0\rangle \otimes |0\rangle \otimes |1\rangle &= |001\rangle = |1\rangle \\
 |0\rangle \otimes |1\rangle \otimes |0\rangle &= |010\rangle = |2\rangle \\
 |0\rangle \otimes |1\rangle \otimes |1\rangle &= |011\rangle = |3\rangle \\
 |1\rangle \otimes |0\rangle \otimes |0\rangle &= |100\rangle = |4\rangle \\
 |1\rangle \otimes |0\rangle \otimes |1\rangle &= |101\rangle = |5\rangle \\
 |1\rangle \otimes |1\rangle \otimes |0\rangle &= |110\rangle = |6\rangle \\
 |1\rangle \otimes |1\rangle \otimes |1\rangle &= |111\rangle = |7\rangle
 \end{aligned}$$



# The Computational Basis

- Each additional qubit **doubles** the number of possible states.
- For a 2 qubit system  $|00\rangle, |10\rangle, |01\rangle, |11\rangle$
- Adding one qubit doubles the number of possible states.
- For a 3 qubit system:  $|000\rangle, |100\rangle, |010\rangle, |001\rangle, |101\rangle, |011\rangle, |110\rangle, |111\rangle$
- Consequently, representing a system with  **$n$  qubits requires on the order of  $2^n$  classical bits.**



*There's an ancient story on the origins of chess that shows the power of doubling. Start with 1 grain of rice, then doubled it each time on the next square. What do you need at the end?*

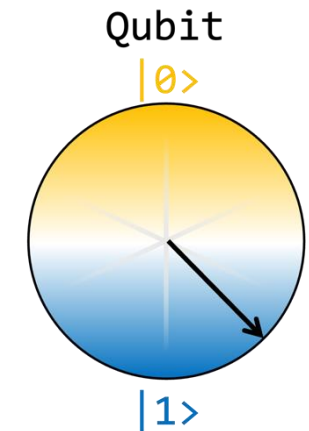
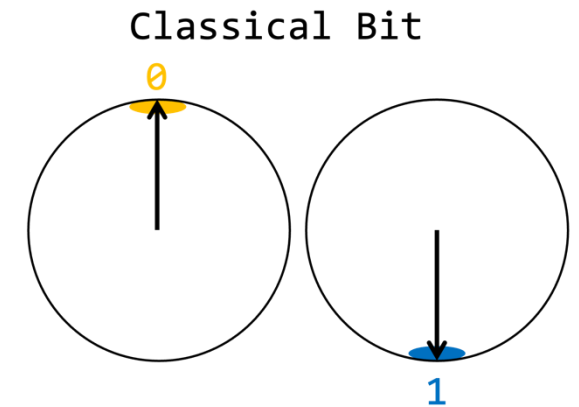
**The last square needs 18,446,744,073,709,551,615 grains, weighing the same as 2000 billion people!**

# State-Vectors: Superposition

- The state of a quantum computer is given by its state-vector  $|\psi\rangle$ .
- $|\psi\rangle$  can be expressed as a **superposition** (linear-combination), of computational basis states.
- A system of qubits is finite-dimensional so we can always represent  $|\psi\rangle$  as a **column vector**.
- The possible states for a **single** qubit:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

- Coefficients  $\alpha$  and  $\beta$  are complex numbers with a **magnitude** and **phase**.
- A qubit can store information in the magnitude and phase of its coefficients – **more information than a classical bit**.



# State-Vectors: Superposition

- Putting together two qubits:

$$|\psi\rangle = (\alpha_0|0\rangle + \beta_0|1\rangle) \otimes (\alpha_1|0\rangle + \beta_1|1\rangle)$$

$$= \alpha_0\alpha_1|00\rangle + \alpha_0\beta_1|01\rangle + \beta_0\alpha_1|10\rangle + \beta_0\beta_1|11\rangle$$

$$= \alpha_0\alpha_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \alpha_0\beta_1 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \beta_0\alpha_1 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \beta_0\beta_1 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Superposition

## State-Vectors: Interference

- Differences in the complex **phase** of  $\alpha_i$  and  $\beta_i$  cause **constructive or destructive interference**, altering the magnitude of basis state in the superposition.

$$|\psi\rangle = (\alpha_0|0\rangle + \beta_0|1\rangle) \otimes (\alpha_1|0\rangle + \beta_1|1\rangle)$$

$$= \alpha_0\alpha_1|00\rangle + \alpha_0\beta_1|01\rangle + \beta_0\alpha_1|10\rangle + \beta_0\beta_1|11\rangle$$

$$= \alpha_0\alpha_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \alpha_0\beta_1 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \beta_0\alpha_1 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \beta_0\beta_1 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Interference

# State-Vectors: Measurement

- Quantum computing relies on quantum superposition—a single quantum object can occupy multiple states simultaneously.
- This phenomenon has been extensively experimentally confirmed.
- Equally well-established is the phenomenon of **collapse**.

When we **measure** the state of a quantum computer, we obtain **one** bit string that corresponds to just **one** of an exponential number of possible basis states.



# State-Vectors: Measurement

- But which state does it collapse to?
- The coefficients of  $|\psi\rangle$  are **probability amplitudes** for the corresponding basis states.
- To compute the probability of measuring a particular state  $|k\rangle$ , we inner-product  $|\psi\rangle$  with its corresponding **column vector**  $\langle k|$ , compute the complex modulus and square the result:

$$\begin{aligned} \text{Prob}(|01\rangle) &= |\langle 01|\psi\rangle|^2 = \left| [0 \quad 1 \quad 0 \quad 0] \cdot \left( \alpha_0\alpha_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \alpha_0\beta_1 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \beta_0\alpha_1 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \beta_0\beta_1 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right) \right|^2 \\ &= |\alpha_0\beta_1|^2 \end{aligned}$$

- **Note 1:** As the state-vector fully-describes a quantum system:

$$|\alpha_0\alpha_1|^2 + |\alpha_0\beta_1|^2 + |\beta_0\alpha_1|^2 + |\beta_0\beta_1|^2 = 1$$



# State-Vectors: Measurement

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- **Note 2:** Computational basis states are **orthonormal**:

$$\langle k|k'\rangle = \begin{cases} 1, & \text{if } k = k' \\ 0, & \text{otherwise} \end{cases}$$

# State-Vectors: Entanglement

- Superposition interference and collapse can together give rise to **entanglement**.
- In an entangled state, a measurement outcome on one qubit determines the possible outcomes of another.
- Famously referred to as “spooky action at a distance” by Einstein – it is a uniquely quantum phenomena.

A two-qubit “Bell state”:

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

- Computationally, entanglement enables “controlled” operations between superposed basis states. Analogous to classical “if then” statements.



# Unitaries and Quantum Circuits

- $|\psi\rangle$  is manipulated by **unitary operators**  $U$
- $U$  transforms  $|\psi\rangle$  while ensuring that its measurement probabilities still sum to one.
- A unitary operator for a system with  $n$  qubits can be represented as a  $2^n \times 2^n$  matrix.
- By definition, the inverse of any  $U$  is its conjugate transpose (or “adjoint”).
- So, we can always retrieve the initial state vector:

$$|\psi'\rangle = U|\psi\rangle$$

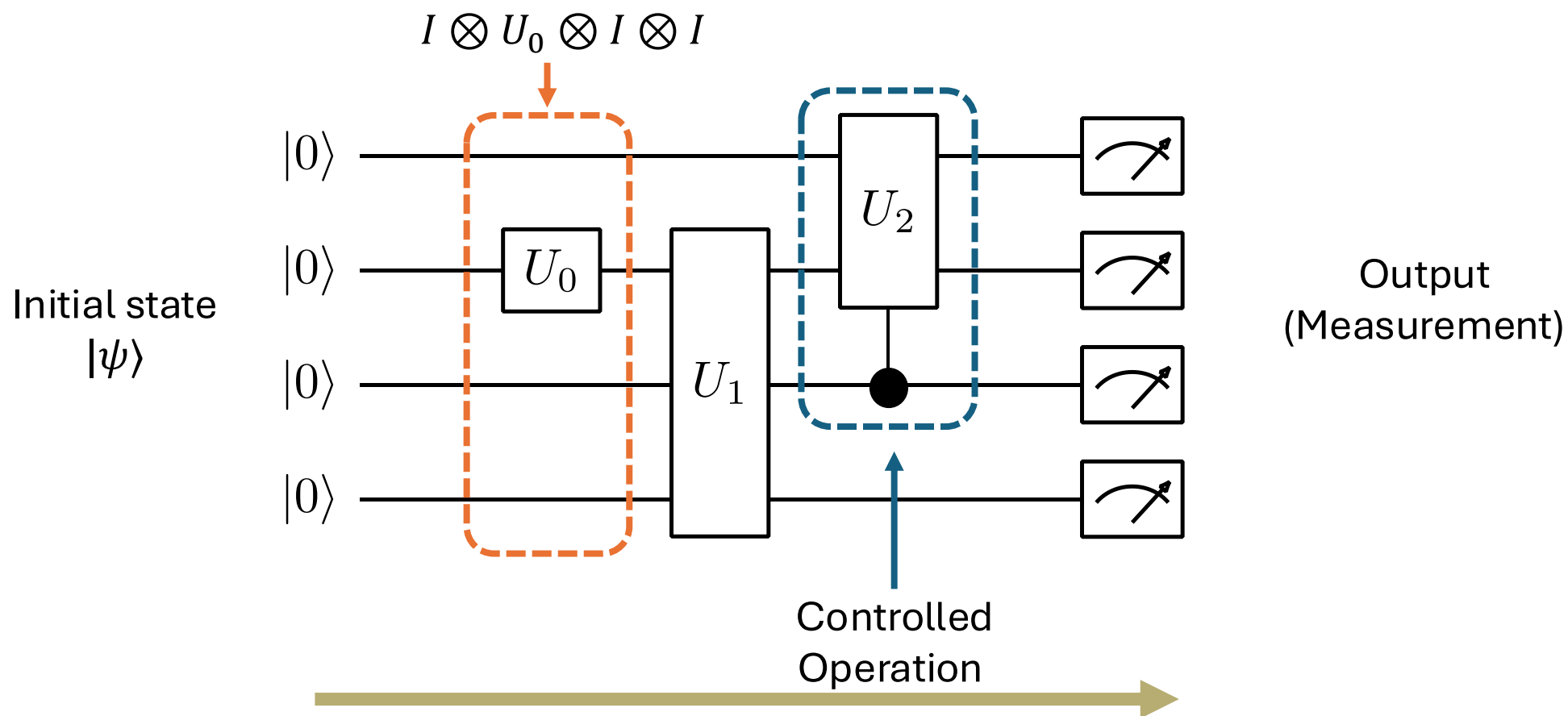
$$|\psi\rangle = U^{-1}|\psi'\rangle$$

- Quantum computing is a type of **reversible** computation.



# Quantum Circuits

- Quantum circuits are a diagrammatic representation of a quantum program.
- They show how unitaries are applied to “registers” of qubits and where measurement occurs.

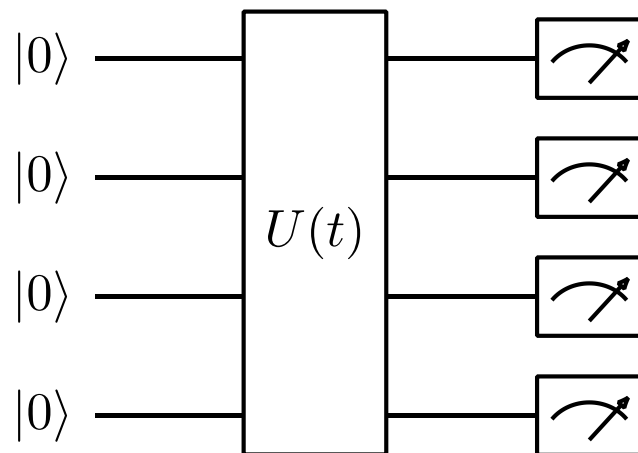


# Quantum Computing Models: Analogue

- Continuously manipulates quantum states through engineered interactions between particles (e.g. atoms or ions). Viewed as applying a time-dependent unitary to prepare a target state:

$$U(t)|\psi\rangle = |\psi'\rangle$$

- Relies on the physical characteristics of the system and is less flexible for general-purpose computation.
- Applications include optimisation, graph-based problems and reservoir computing.

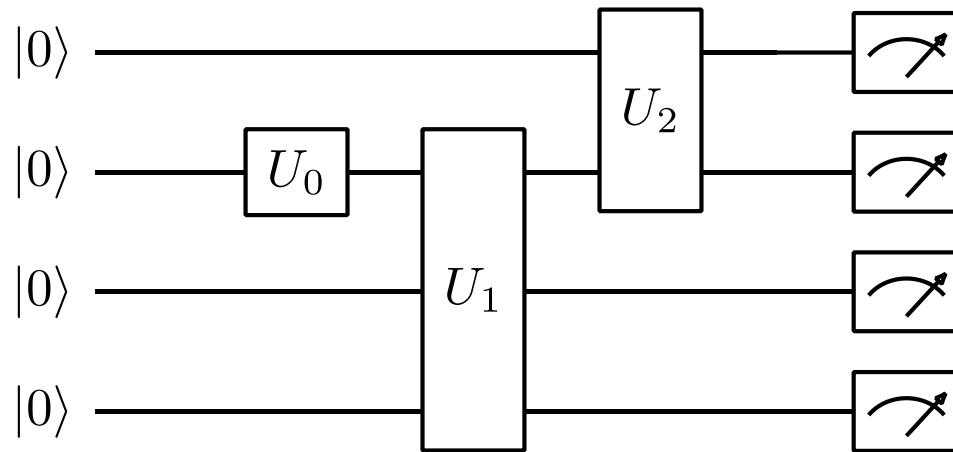


# Quantum Computing Models: Digital

- Uses discrete **quantum gates** - analogous to classical logic gates. Viewed a sequence of unitaries, called quantum “gates”, that act on one or more qubits:



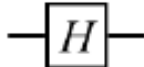
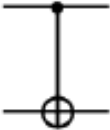
$$U_2 U_1 U_0 |\psi\rangle = |\psi'\rangle$$

- Digital systems are more versatile and are the foundation for universal quantum computers.
- More dependent on error mitigation or correction.


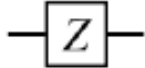
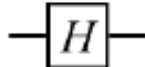
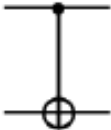


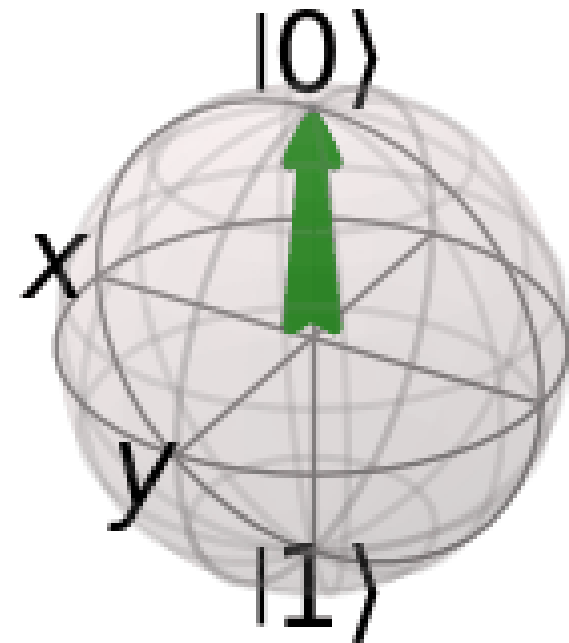
Gate	Notation	Matrix
NOT (Pauli- $X$ )		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli- $Z$		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard		$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
CNOT (Controlled NOT)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

# Quantum Computing Models: Digital



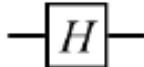
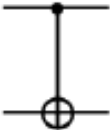
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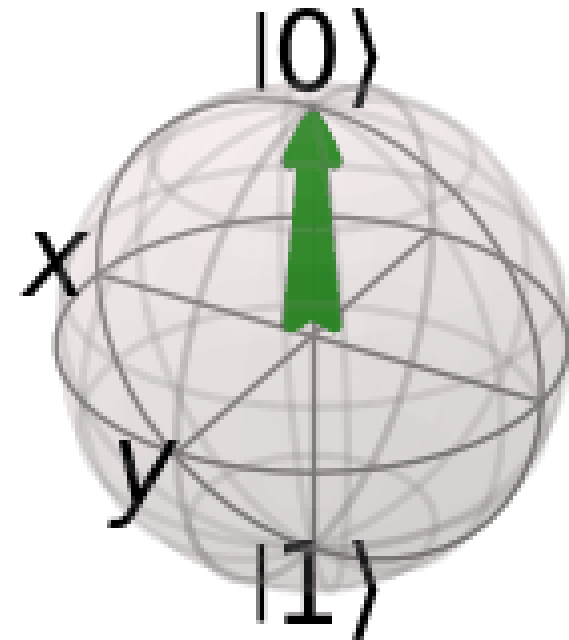
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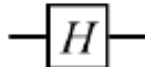


# Quantum Computing Models: Digital

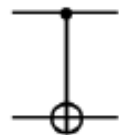
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CNOT  
(Controlled NOT)



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\text{CNOT } |00\rangle = |00\rangle$$

$$\text{CNOT } |01\rangle = |01\rangle$$

$$\text{CNOT } |10\rangle = |11\rangle$$

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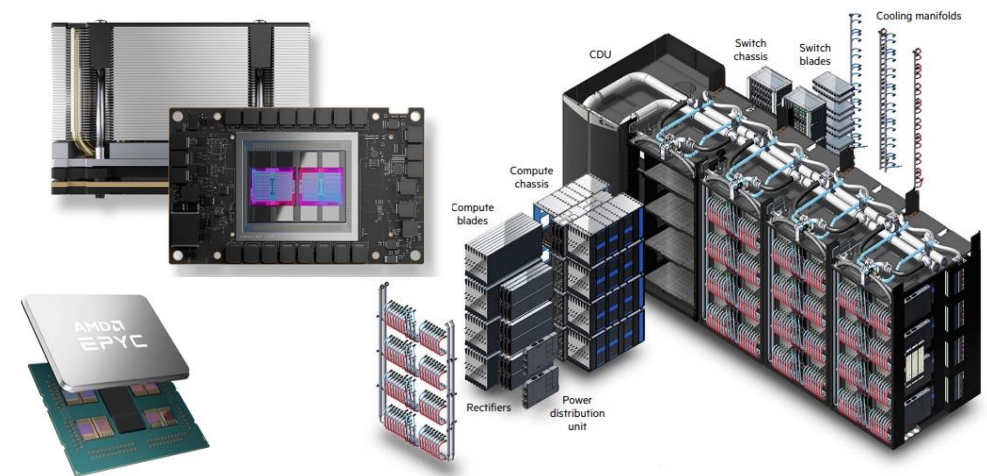
Control

Target



# Classical computers

- Stores bits to represent data, applies bit manipulation to represent operations
- Really fast (GHz)
- Good at arithmetic,  $2 + 2 = 4$  (almost) always
- Production scale, with systems relying on silicon transistors.
- Some variety in hardware but all pretty similar.

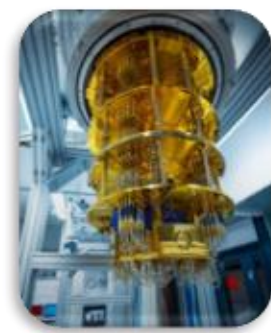
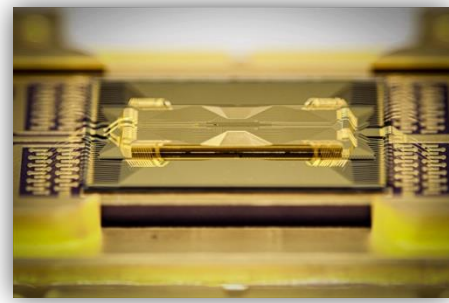


# VS

# Quantum computers



- Operates on “qubits”
- Not good at arithmetic BUT (much) better at other problems.
- We are in Noisy Intermediate Scale Quantum (NISQ) Era.
  - Current technologies are all noisy (90-99% fidelity on single gate operations).
  - Low qubit counts (100)
  - Quite slow (100 circuits per second)
- Lots of variety in technologies, all single experiments



# Quantum Computing Hardware

## Analogue

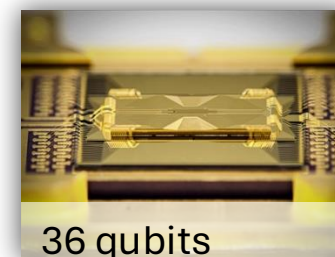
- Superconducting Flux Qubits: D-Wave Systems
  - Needs Helium 3 cooling
- Neutral Atoms: QuEra Computing, Pasqal, Infleqtion
  - Needs vacuum chamber, lasers



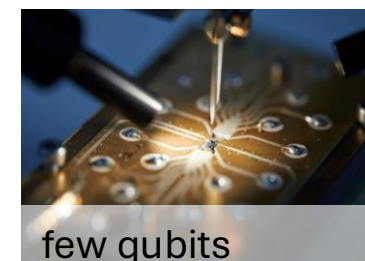
20-50 qubits



130-150 qubits



36 qubits

256 qubits,  
analog computer

few qubits

# Quantum Computing Hardware

## Digital

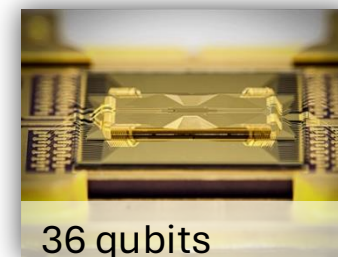
- Superconducting Qubits: IQM, IBM, Google Quantum AI, Rigetti Computing
  - Needs Helium 3 cooling
- Trapped Ions: IonQ, Quantinuum
  - Needs vacuum chamber, magnetic traps, lasers
- Neutral Atoms: QuEra Computing, Pasqal, Infleqtion
  - Needs vacuum chamber, magnetic traps, lasers
- Silicon Spin Qubits: Silicon Quantum Computing (SQC), Diraq
  - Needs Helium 3 cooling



20-50 qubits



130-150 qubits



36 qubits

256 qubits,  
analog computer

few qubits

# Quantum Computing Hardware

## Digital (cont.)

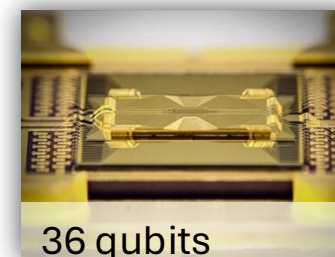
- Diamond-Based Qubits: Quantum Brilliance
  - Currently limited scaling
- Photonic Qubits: ORCA, Xanadu
- Silicon Photonic Qubits: PsiQuantum
  - Needs Helium 3 cooling



20-50 qubits



130-150 qubits

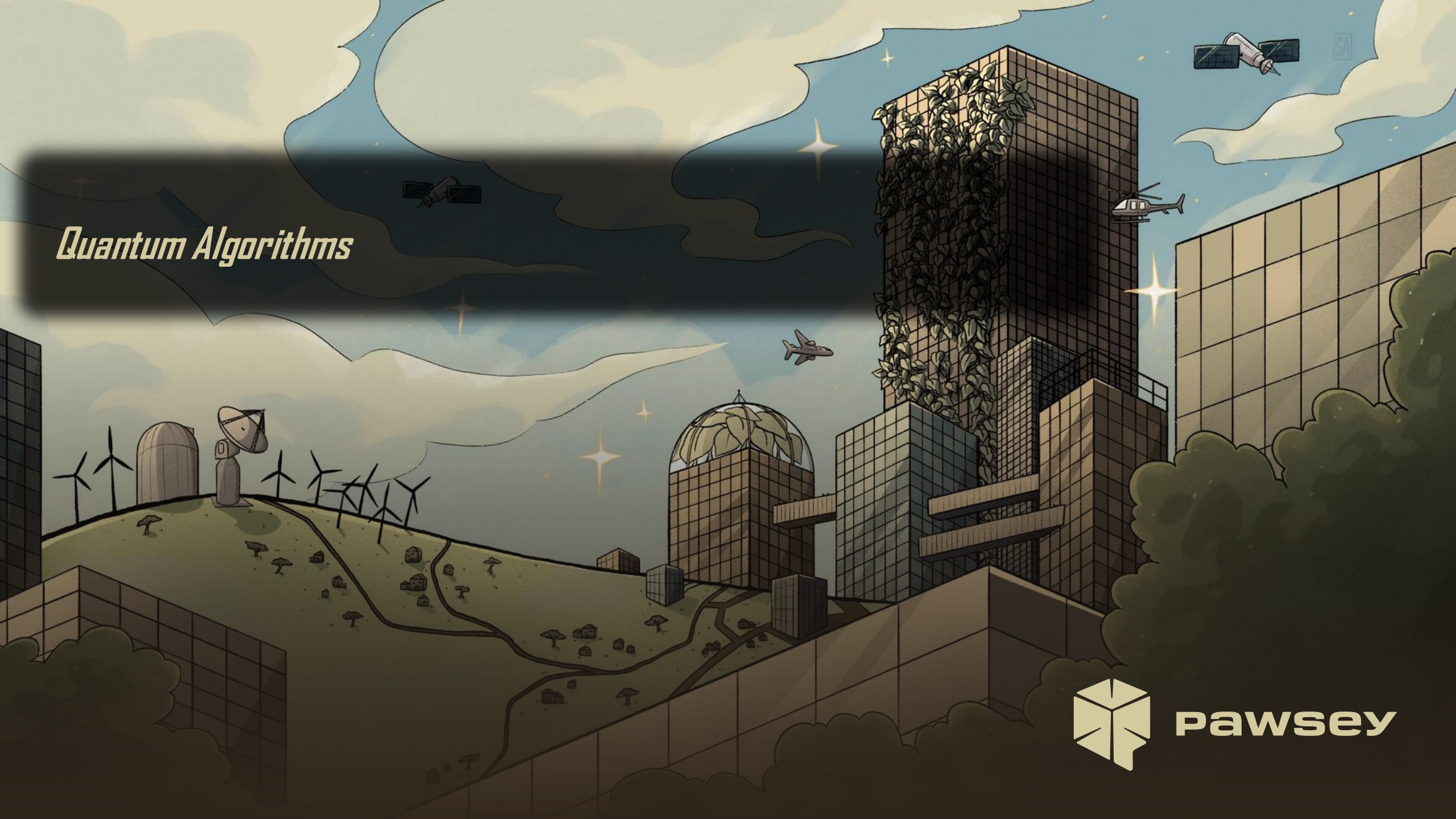


36 qubits

256 qubits,  
analog computer

few qubits

*Quantum Algorithms*



**pawsey**

# Quantum Parallelism

- Consider a bijective function on binary strings of length  $n$ ,

$$f: \{0,1\}^n \rightarrow \{0,1\}^n.$$

- The domain of  $f$  is **exponentially large**, with  $N = 2^n$  elements.
- We can model this function on a quantum computer by a unitary  $U$  that acts on computational basis states  $|x\rangle$  as,

$$U|x\rangle = |f(x)\rangle.$$

- Let  $|s\rangle$  be a **uniform superposition** over all  $n$ -qubit  $|x\rangle$ ,

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle.$$

- We can evaluate  $f$  on its entire domain in one step,

$$U|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |f(x)\rangle.$$

# Quantum Parallelism

- **Quantum parallelism** through quantum superposition underpins the potential for quantum algorithms to provide a speedup over classical methods.

# Quantum Parallelism

- **Quantum parallelism** through quantum superposition underpins the potential for quantum algorithms to provide a speedup over classical methods.

**Beware: Quantum parallelism alone does not provide an algorithmic advantage.**

We also require:

- Implementation of  $U$  using a non-exponential number of qubits and gates.
- Manipulation of  $U|x\rangle$  so useful information can be retrieved with a small number of measurements.



# Adiabatic Quantum Computing

Adiabatic Quantum Computing (AQC) is an analogue method commonly used for combinatorial optimisation (e.g. logistics problems or Ising models).

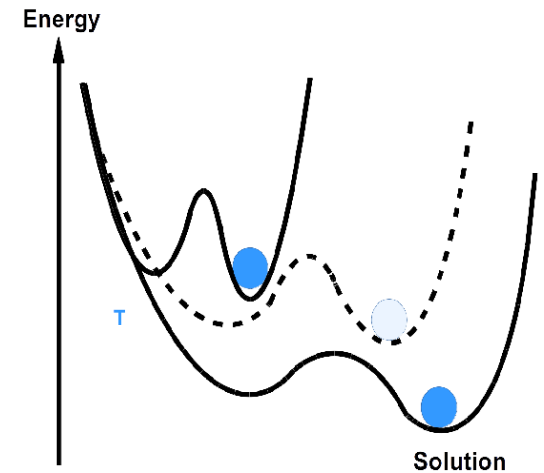
The Method:

1. Define an initial Hamiltonian  $H_0$  that is easy to prepare on a given device
2. Define a final Hamiltonian  $H_1$  that encodes the objective function  $f(x)$ , with optimal solutions corresponding to low-energy/ground states:

$$H_1 = \sum_{x=0}^{N-1} f(x)|x\rangle\langle x|$$

3. Start in  $H_0$

*(cont.)*



Adiabatic evolution

A **Hamiltonian** is an operator that describes the energy of a quantum system and determines how it evolves over time.

# Adiabatic Quantum Computing

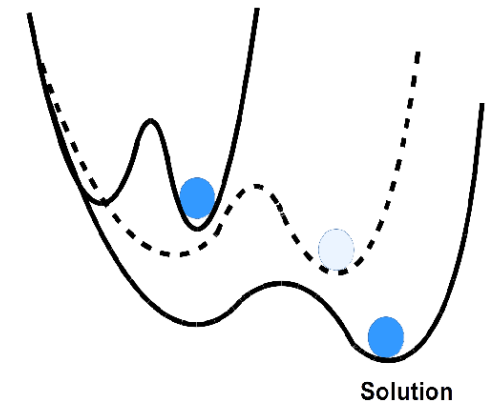
(cont.)

3. Slowly evolve to  $H_1$  via

$$H(t) = (1 - s(t))H_0 + s(t)H_1, \quad s(t): 0 \rightarrow 1$$

4. If evolution is sufficiently slow, the system remains in the ground state and measurement reveals the solution.

- Quantum Annealers (e.g., D-Wave) are specialised devices for AQC.
- Challenges associated with scaling of the annealing time with problem size.

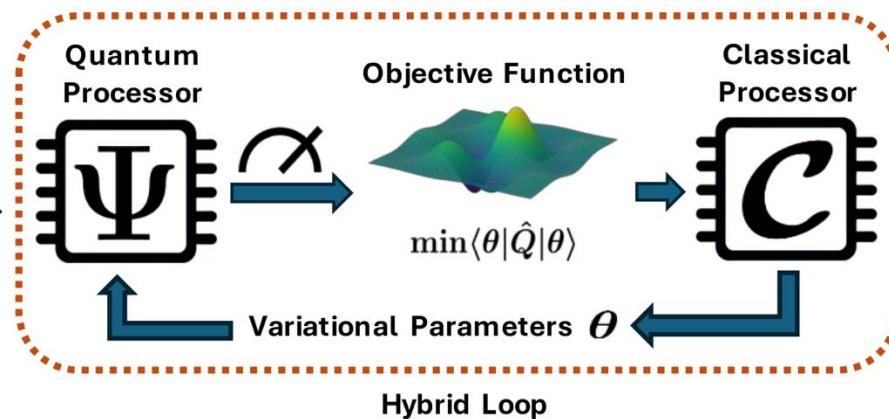
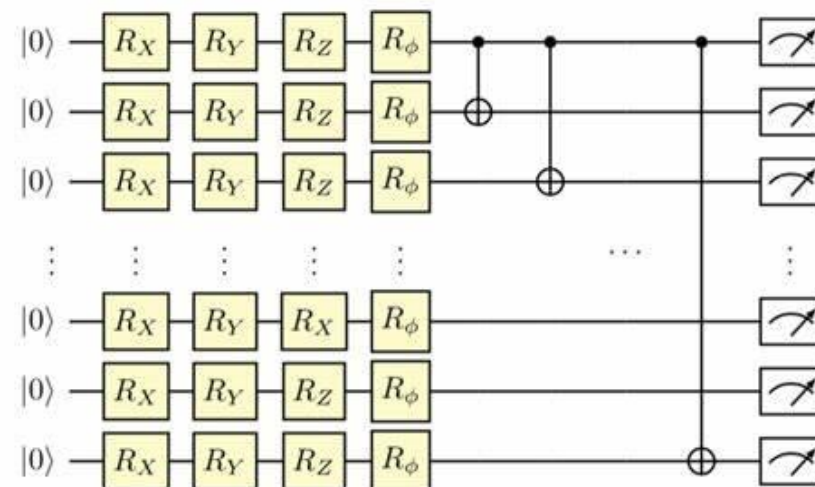


Adiabatic evolution

A **Hamiltonian** is an operator that describes the energy of a quantum system and determines how it evolves over time.

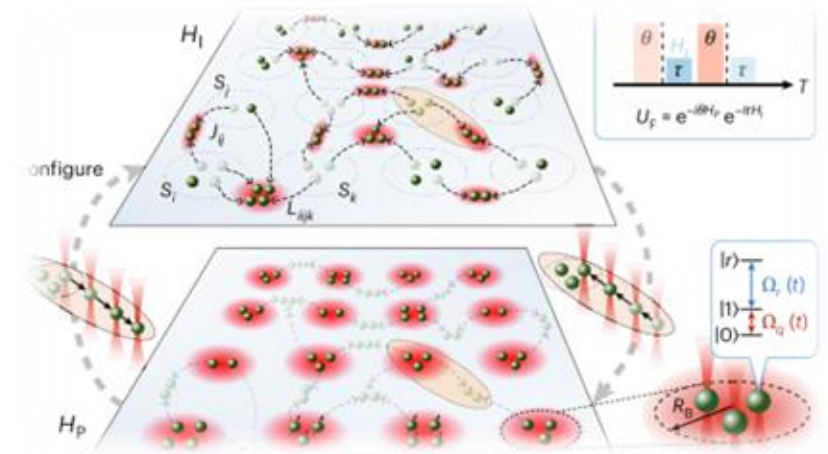
# Variational Quantum Circuits

- Quantum circuit with adjustable parameters (e.g. angles in rotation gates).
- Built up in layers, the depth of the circuit is adjusted to match problem complexity and hardware constraints.
- Measurement and feedback loop:
  1. Quantum circuit prepares a state
  2. Measure to compute an objective function
  3. A *classical optimiser* updates parameters
  4. Repeat until convergence.
- Algorithms for combinatorial optimisation, eigenvalue solving and machine learning fall within this model.
- These are a form of hybrid quantum-classical computation.



# Hamiltonian Simulation

- Compute  $|\psi(t)\rangle = \exp(-itH)|\psi(0)\rangle$
- Applications quantum chemistry, condensed matter physics, and materials science—some of the most anticipated real-world applications.
- Takes advantage of memory-efficient representation of the Hamiltonian using qubits.
- Can be performed natively on analogue quantum computers by tuning the hardware's Hamiltonian to match or approximate a Hamiltonian of interest.
- Can be performed on digital quantum computers – to varying degrees of accuracy and efficiency depending on the chosen method and properties of  $H$  (as with classical computers).



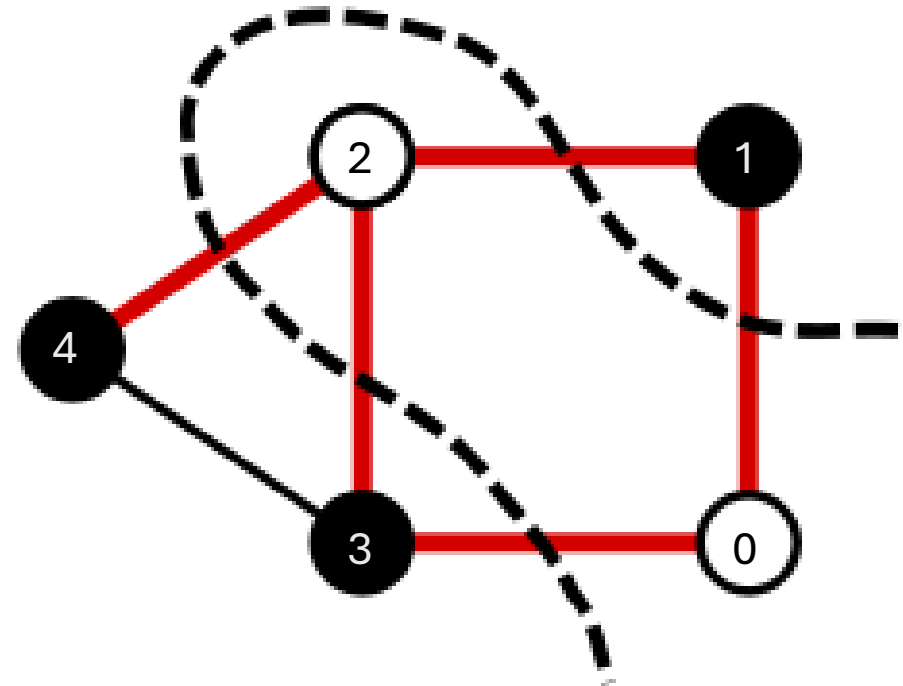
# The Max-Cut Problem

## Problem Statement:

- For a given graph, find an assignment of vertices to two sets such that the number of adjacent vertices in different sets is maximised.

## Why do we care?

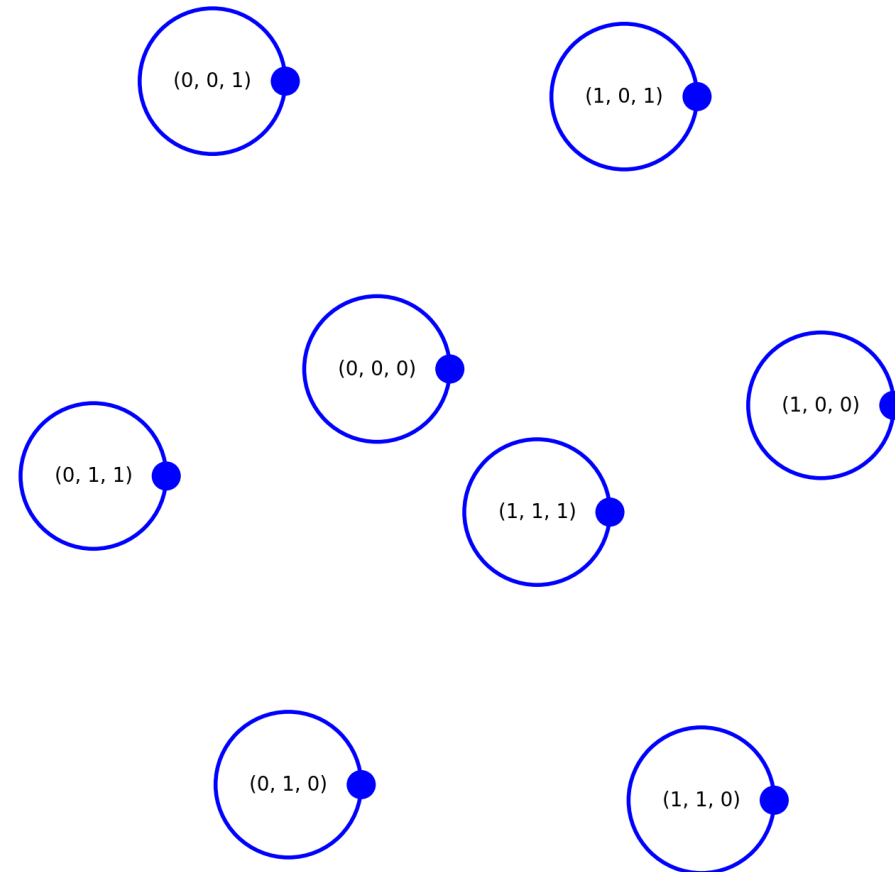
- A simple to state problem that is computationally hard (NP-hard).
- Other difficult optimisation problems can be framed as a max-cut problem: electrical circuit layouts, spin models, portfolio optimisation...
- Straight-forward to problem express on a quantum computer.



Solution:  $\mathbf{z} = 10110$   
 Cut-value:  $C(\mathbf{z}) = 5$   
 Encoding:  $\mathbf{z} \rightarrow |\mathbf{z}\rangle = |10110\rangle$

# The Quantum Approximate Optimisation Algorithm

- The Quantum Approximate Optimisation Algorithm (QAOA) aims to find optimal or near-optimal solutions to combinatorial optimisation problems. We apply it here to the MaxCut problem.
- Possible are mapped to the basis states of qubits. Each basis state is a possible cut.
- A variational quantum circuit “**ansatz**” prepares a state where the probability of measuring large-cut solutions are amplified.

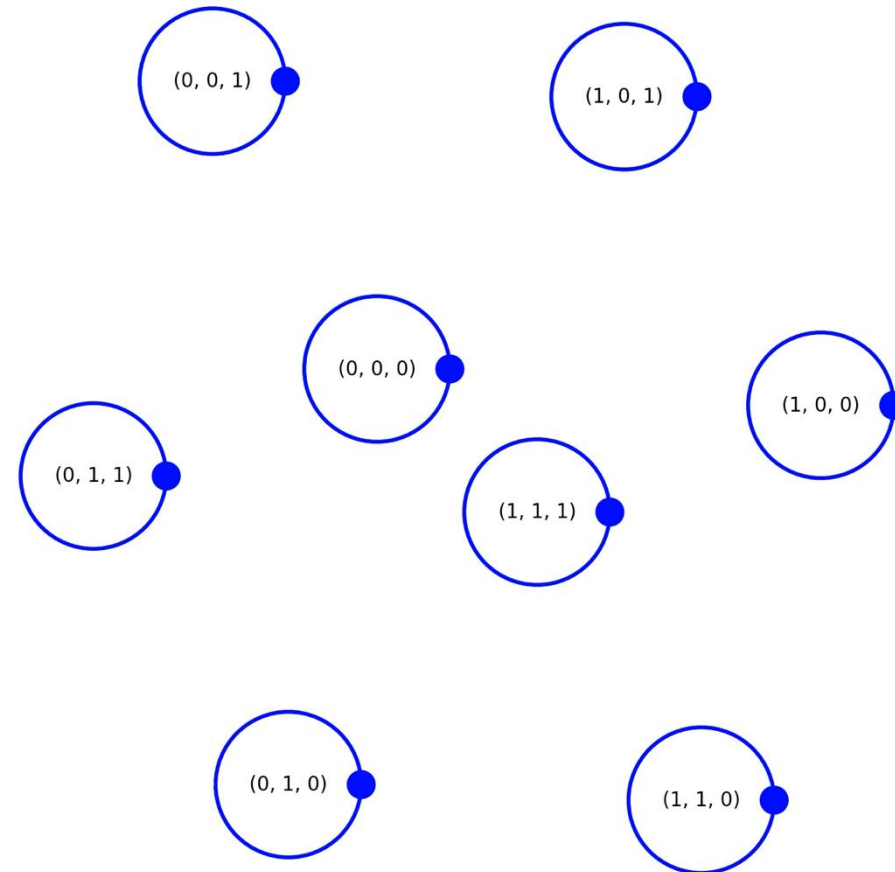


# The Quantum Approximate Optimisation Algorithm

- The quantum circuit of the QAOA has three main components.
- **First:** preparation of a uniform superposition over all possible solutions/basis states.
- **Second:** a phase-shift unitary rotations the phase of each state proportionally to its assigned cost.
- For the max-cut problem the rotation angle is

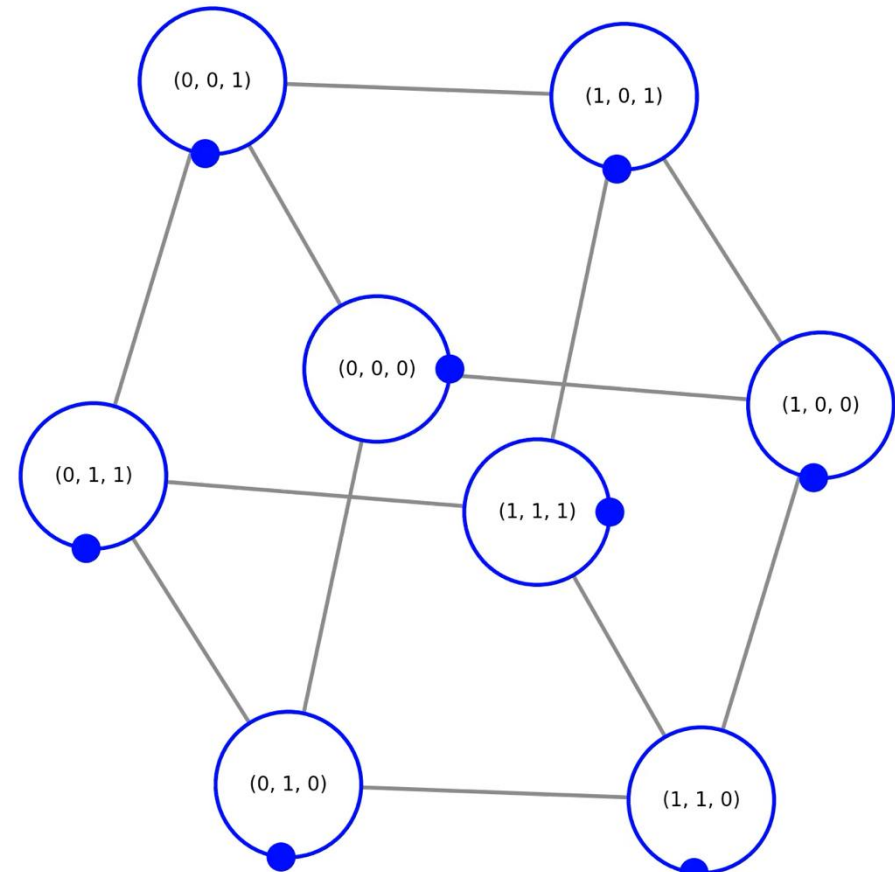
$$\gamma C(\mathbf{z})$$

- $\gamma$  is a variational parameter and  $C(\mathbf{z})$  is the value of the cut for solution  $\mathbf{z}$ .



# The Quantum Approximate Optimisation Algorithm

- **Third:** a mixing-unitary drives the transfer of amplitude between basis states.
- This is typically achieved by applying a Pauli-X rotation to each qubit for angle  $\beta$ , where  $\beta$  is another variational parameter.
- During this step, phase differences due to the phase-encoded cut-values cause destructive and constructive interference.
- The phase-shift and mixing unitary are applied for  $p$  iterations. As  $p$  increases, so does the degree of possible interference.



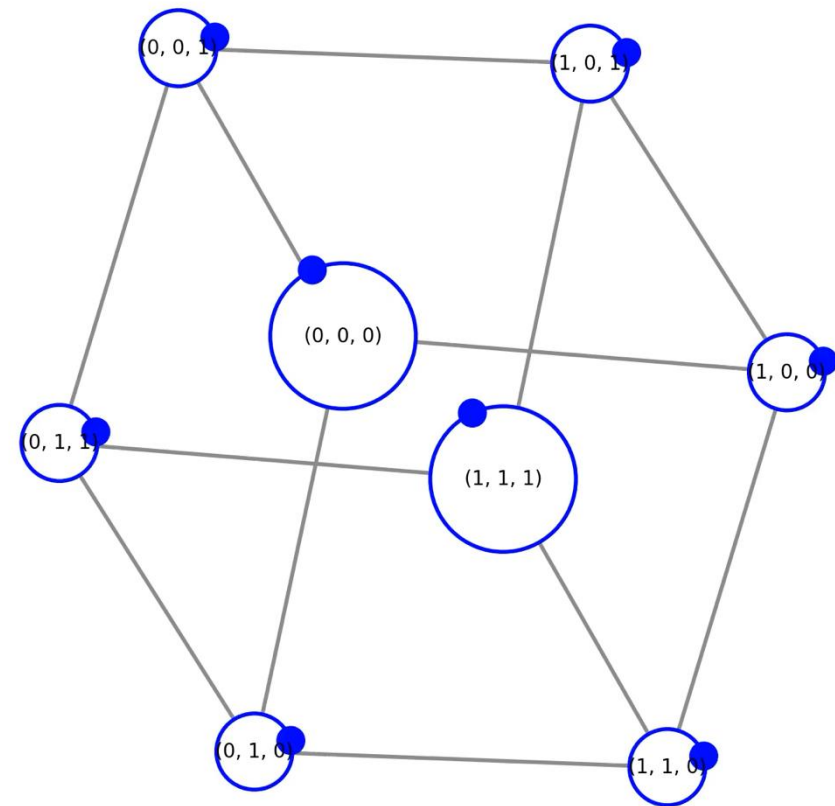
# The Quantum Approximate Optimisation Algorithm

- The full set of variational parameters is:

$$\gamma = (\gamma_1, \dots, \gamma_p)$$

$$\beta = (\beta_1, \dots, \beta_p)$$

- These are tuned by a classical optimiser whose objective function is the average measured (expectation value) of the cut-values.
- As the objective function increases, so does the probability of measuring an optimal, or near-optimal, solution.



# Conclusion

- Quantum computing is progressing rapidly, but is still not production-ready.
- Current efforts focus on near-term systems: how to construct and use them effectively.
- Computing is a broad, collaborative field—quantum computing is no different, drawing on physics, math, engineering, and beyond.
- You can get involved: domain-specific applications (beyond physics) are driving interest, while tools from theoretical physics are increasingly being repurposed for new algorithmic insights.

# Workshop Activity: Hands-On Quantum Computing

**[https://github.com/PawseySC/PHYS4004\\_quantum\\_computing](https://github.com/PawseySC/PHYS4004_quantum_computing)**